Privacy-aware Regression Modeling of Participatory Sensing Data

Hossein Ahmadi, Nam Pham, Raghu Ganti, Tarek Abdelzaher, Suman Nath, Jiawei Han
hahmadi2@uiuc.edu, nampham2@uiuc.edu, rganti@us.ibm.com, zaher@uiuc.edu, sumann@microsoft.com, hanj@uiuc.edu

Abstract
Many participatory sensing applications use data collected by participants to construct a public model of a system or phenomenon. For example, a health application might compute a model relating exercise and diet to amount of weight loss. While the ultimately computed model could be public, the individual input and output data traces used to construct it may be private data of participants (e.g., their individual food intake, lifestyle choices, and resulting weight). This paper proposes and experimentally studies a technique that attempts to keep such input and output data traces private, while allowing accurate model construction. This is significantly different from perturbation-based techniques in that no noise is added. The main contribution of the paper is to show a certain data transformation at the client side that ensures that no noise is added. The main contribution of the paper is that no noise is added. The main contribution of the paper is significantly different from perturbation-based techniques in that no noise is added. The main contribution of the paper is to show a certain data transformation at the client side that helps keeping the client data private while not introducing any additional error to model construction. We particularly focus on linear regression models which are widely used in participatory sensing applications. We use the data set from a map-based participatory sensing service to evaluate our scheme. The service in question is a green navigation service that constructs regression models from participant data to predict the fuel consumption of vehicles on road segments. We evaluate our proposed mechanism by providing empirical evidence that: i) an individual data trace is generally hard to reconstruct with any reasonable accuracy, and ii) the regression model constructed using the transformed traces has a much smaller error than one based on additive data-perturbation schemes.

K.4.1 [Computing Milieux]: Computers and Society—Privacy; G.0 [Mathematics of Computing]: General

General Terms
Algorithms, Design, Experimentation

Keywords
Privacy, Participatory sensing, Linear regression

1 Introduction
This paper develops a privacy-aware scheme for sharing participatory sensing data towards the computation of generalizable models from the data pool. We consider applications where construction of such models requires data that are private, whereas the models themselves are not. For example, individuals might not want to share their weight, food intake, and exercise habits on daily basis, yet a quantitative model on how food intake, weight loss, exercise, and other lifestyle choices are generally related could be of interest to many.

Participatory sensing applications have recently become a popular trend in providing community-wide services that “crowdsource” the task of data sensing [1, 7, 16, 23]. One can generally distinguish between at least two types of services from the perspective of the end-result computed by the service. The first type represents services where some statistic is computed from input data. For example, the service might compute the quality of air, speed, potholes, or pollen levels on city streets. These statistics pertain only to the neighborhoods where data was collected. It is not the purpose of the service to generalize; the existence of potholes or pollution in one neighborhood does not necessarily entail their existence in a different neighborhood.

In this paper, we consider a different type of participatory sensing applications; one where the collected data are multidimensional and sparse. Hence, the objective is to generalize from data we collected to extrapolate and predict data we do not have. An example might be to collect, in addition to pollution data, some context information as well, such as population density and eco-friendly behaviors of the population. This can then be used to build a model relating these factors to pollution levels and use it to predict pollution elsewhere, where we do not have measured pollution data. We particularly consider linear regression modeling since it has already been employed for similar purposes in participatory sensing applications (e.g., [19]). It is often the case, that the general...
models thus computed are themselves not private, but some
or all inputs of those models are private data.

Many prospective applications fall into this category. For
example, consider a participatory sensing service that mod-
el the energy consumption of a household based on the us-
age of various appliances and the time of year in order to help
users save on the energy costs. Although the constructed
model (for energy consumption) is public, the input and out-
put values contain users’ private information that cannot be
easily shared; a user may not wish to reveal the usage of
the appliances at specific times. This paper focuses on in-
creasing the privacy of individual user data while enabling
community-wide data modeling for prediction and extrap-
olation purposes.

Several options exist for sharing private data. The most
popular include ensuring anonymity of the data source. To
make service architecture simple and boost trust, we instead
allow users to share their identities with the aggregation
server openly but give them means to alter their data in a way
that makes the original values hard to recover. It remains
an open issue whether the average user will be more trust-
ing of security mechanisms that promise to fully anonymise
their identity or data alteration mechanisms that promise to
irrecoverably alter their data. One advantage of the latter
is that it can be entirely implemented on the client side,
whereas some of the former need support from other nodes
as well. Whether the perceived difference in the needed trust
base is of consequence to the average user remains to be
seen, which is an argument for exploring both types of ap-
proaches and letting deployment experience decide on their
relative usability advantages. This paper investigates the data
alteration approach.

Additive data perturbation (noise) is the most common
way to alter data for privacy [4, 17, 20, 34]. The main idea
behind the perturbation is to mask each individual user data
with noise in such a way that the noise cancels out or can be
decomposed in aggregate calculations. However, error is always introduced to the modeling process depending on
the number of data streams and type of the application. Our
approach is different in that it aims to introduce no additional
error in modeling, regardless of the number of users or data
streams in the system. We should note that the perturbation
schemes apply to a more general class of models than linear
regression models, as long as the noise distribution follows
closely the user data distribution.

We transform the sensitive data at the client side to a set
of aggregate features that usually do not reveal much infor-
mation on original data. Community-wide models that are
computed from these features are exactly the same as the
models computed from the original community data. Users
can control their degree of privacy by controlling the amount
of aggregation involved in feature computation. To compute
the features, time-series data are first segmented based on
some prior knowledge about the system. The segmentation
is on intervals long-enough to eliminate short-scale dynam-
ics of the system. This way, the system can be represented as
an approximately static (time-independent) relation between
various attributes of a data segment. Next, specific feature
matrices are extracted from the data segments. They repre-

Figure 1. A privacy-aware participatory sensing model.

sent the information shared by each user with the commu-
nity. Although our paper lacks formal proofs of its privacy
properties, it presents intuitions and empirical evidence on
why it is often hard to reverse the process and recover pri-
vate user data based on the information shared.

We evaluate our technique using a sensory data set [21]
collected for a green navigation service from current litera-
ture [19]. This map-based service enables finding the most
fuel efficient path between two points for a specific car. The
service relies on data collected from vehicles equipped with
a diagnostic fuel-efficiency scanner device. Results suggest
that in most cases user data cannot be recovered without sig-
nificant error, while model reconstruction is always correct.

The rest of this paper is organized as follows: We present
the motivation and the exact definition of the problem in Sec-
tion 2. Section 3 describes the detailed steps involved in the
privacy framework. In Section 4, we present the computation
of a linear regression model using the data shared by users.
We show that our proposed scheme does not change model-
ing error compared to model construction from the original
raw private data. We discuss the privacy protection of such
transformations in Section 5. In Section 6, we evaluate the
privacy achieved by our technique using a case study based
on the data set from a green navigation service. Section 7
discusses the potential cases leading to privacy breaches and
open questions regarding the approach. We discuss related
work on privacy in participatory sensing applications in Sec-
tion 8. Finally, the paper concludes with Section 9.

2 Problem Formulation

Many participatory sensing applications calculate com-
munity statistics using data shared by the community [7].
Previous privacy-preserving approaches mostly focused on
deriving community statistics while keeping individual data
traces private [20, 34]. This paper considers a different ca-
category of participatory sensing applications where the appli-
cation extrapolates and predicts unavailable data using models
of the system.

Given the nature of applications we target, the follow-
ing summarizes the assumptions and requirements of our
scheme:

- The model relating user inputs to the outputs is public.
- Each data sample collected by an individual is private
  and may not be revealed.
- The models used in the service are linear in coefficients.
- The time-series data can be packed into uncorrelated
data samples by aggregation (over time for example).
2.1 Modeling in Participatory Sensing Applications

Consider an example participatory sensing service aimed to reduce household energy consumption. Each user collects the following data traces daily: i) the total time that various appliances are in use (e.g., TV), ii) the temperature inside the house, and iii) the outside temperature. By sharing the data, a regression model can be constructed and used to predict the energy consumption for a given usage pattern and season. Eventually, the service helps other residents to save on energy costs by adapting their appliance usage.

Figure 1 illustrates our model of privacy-aware participatory sensing applications that construct a model of user data. In our architecture, data traces collected by participants are transformed on the client side to neutral features. The data can typically be divided into a variable we are interested in (e.g., household energy consumption) and other variables we believe are good predictors of the former. The objective is to compute a model relating the predictors (model input) to the variable of interest (model output). The neutral features computed from user data do not easily reveal user data and can thus be made available to any entity that needs them.

The process of attempting to compute the private data from the features is called reconstruction. An important question is: given the information that each user sends to the service, how accurately it is possible to reconstruct the values in the original data traces? To measure privacy, we calculate the sum of squared difference between each value in the original trace and the corresponding value in the reconstructed trace. A lower value means that the reconstructed trace is more similar to the original one and reveals more private information about the user.

The participatory sensing service includes a server that collects the neutral features of all participants to compute the regression model of the underlying measured phenomenon. The model has a predictive property and can thus be of interest to a broad population besides the participants who contributed data to its construction.

2.2 Design Goals and Metrics

There are two challenges in designing a system according to our model. First, we need a scheme that converts the data traces to features that reduces reconstruction accuracy of the data traces. Second, we need to perform the modeling using the shared features instead of the data traces themselves. This immediately leads to the two objectives for the system:

- Minimize the modeling error: It is desirable to reach the same level of modeling accuracy as one attained in a system not employing any data alterations.
- Maximize the reconstruction (breach) error: The higher the error in reconstruction of individual user data, the more it is ensured that the privacy of the user is not breached.

We define a perfect privacy-enabling data sharing scheme in participatory sensing applications to be a scheme that satisfies two conditions:

- Perfect modeling: Model construction from shared data produces exactly the same model as if the original private data traces were used.
- Perfect neutrality: Reconstruction of private user data from shared data yields the same error as if no additional information was available to the outside world (i.e., to an attacker) besides the computed public phenomenon model. Note, however, that the neutrality condition does not exclude information leaks that result from computing the public model itself. For example, if the model suggests that all adults in some population are between 5 and 6 feet high, then something is leaked about individual user data in that population. A perfectly neutral scheme should not introduce any additional leaks.

This paper presents and empirically evaluates a privacy-aware scheme for applications that aim to compute a linear regression model of some measured phenomenon. Our scheme satisfies the perfect modeling condition while empirical evidence suggests that it is also neutral in many cases.

3 Privacy Filter

A participant in a participatory-sensing application uses various sensing devices to collect data samples about a phenomenon (e.g., a thermometer to measure temperature inside a house, a GPS device to measure location, or an OBD-II port to measure vehicular fuel consumption). A privacy filter converts such data to features to be used for phenomenon modeling. In order to attain the goals of a perfect privacy-enabling scheme, our approach is first to convert the traces into a set of uncorrelated samples, we call segments. This is to foil correlation-based attacks. Clients then report neutral features computed over a set of segments. They are used later for model construction.

This section details the various algorithmic steps involved on the client side and presents the structure of the features which are sent to the server. Later in Section 4 we describe how our scheme achieves the perfect modeling property. In Section 5 we present empirical evidence to support some level of neutrality for our scheme and describe how it can be used to personalize the level of privacy provided.

The data collected by a user is usually in form of time-series. For example, a sensing device might record the value of temperature every second. However, the phenomenon model is typically concerned with predicting quantities at longer time-scales. For example, how much gas a vehicle will spend on a given route? How much energy a household will save if they installed motion-activated light controls? How much weight a 300lb person might lose if engaged in a particular diet and exercise routine? At those time-scales, fast system dynamics are averaged out. The aggregate output of a system in a sufficiently large time-interval is more correlated with the aggregate input in that interval and not what happens in other intervals. For example, one’s weight loss in one month might be more correlated with food intake in that month, rather than the month before. This is not necessarily true of fluctuations at shorter timescales. This process of aggregation over appropriate timescales is the first step in our client-side data alteration. It is called data segmentation. The segment aggregate can be the sum (e.g., of consumed energy) or in some cases the average. The main reason to do the segmentation is to eliminate correlations between sam-
ple taken at different times.

This section first describes how the segmentation is done and why the segmented data cannot be shared without possibly violating privacy of the individual. The next step in our scheme is to convert the segmented data into neutral features that are less of a threat to users’ privacy.

3.1 Data Segmentation

In order to perform segmentation on raw traces, we first need to decide on the time interval to use. This often requires application specific knowledge. There are three criteria involved: i) a large enough time interval ensures that the system can be described using a static model and hence remove correlations among samples, ii) the time interval should result in an accurate prediction, and iii) the time interval should be usable by the participatory-sensing service.

In our household energy consumption example, various time intervals such as a day, a month, or a year ensure a static description of the system and so the first property is achieved. Very small time intervals like daily energy consumption usually highlight the effect of modeling noise and therefore fail to achieve the second property. On the other hand, the energy consumption over a year may not be as useful since laws of large numbers may make such averages converge, resulting in ill-condition matrices. Hence, considering the data collected by each user in a given application, as illustrated in the previous example, the data might be segmented into one-month intervals before being shared with the community. Table I presents sample values for such segmentation. The above discussion is to present general guidelines. Actual segmentation period will vary substantially from one application to the next.

The result of the segmentation is a set of \( n \) data points. Each data point consists of \( d \) input values corresponding to model inputs (input dimensions) and a single output value. Note that, by doing the segmentation, we actually remove the time attribute from all of the data and make each data point time-independent. In particular, there is no particular order maintained for the segmented data. We use \( y_i \) to denote the value of the output attribute in the \( i \)-th segment (e.g., energy consumption) and \( x_{ij} \) to denote the value of \( j \)-th input of segment \( i \). Formally, an appropriate time interval for the segmentation ensures that \( y_i \) can be estimated accurately using:

\[
\hat{y}_i = f(x_{i1}, \ldots, x_{id})
\]

Although the data segmentation performs aggregation on the original data trace, sharing raw segments can result in a breach of privacy. For example, Table II shows appliance usage and temperature inside a house each month. Now, these values can easily show whether a residence is occupied or not in a particular month. Therefore, we need to take another step and only share some features of the segmented data that are less likely to threaten privacy. This is presented next.

3.2 Neutral Features

A multi-dimensional linear regression model relates an output variable to several predictor variables or simply inputs. Consider the segmented data from above and the notation used to represent input and output values in each data point. Let \( Y = \{ y_i : 1 \leq i \leq n \} \) and \( X = \{ x_{ij} : 1 \leq i \leq n, 1 \leq j \leq d \} \). We should note that a linear regression model is only linear with respect to the regression coefficients. The predictor variables can be any arbitrary function of the input attributes. Several general purpose regression techniques explore a range of feature spaces (e.g., quadratic features where the predictors are product of two input attributes). To generalize, we use \( w_{ij} = g_j(x_{i1}, \ldots, x_{id}) \) and denote \( W = \{ w_{ij} : 1 \leq i \leq n, 1 \leq j \leq k \} \). A linear regression model of this system describes the system using:

\[
Y = W\eta + \epsilon
\]

where \( \epsilon \) is assumed to be a zero-mean error term with a constant variance \( \sigma^2 \). \( \eta \) is the model coefficient matrix and can be estimated by various regression techniques [29]. Consequently, the output estimate, \( \hat{Y} \), is given using:

\[
\hat{Y} = W\eta
\]

In our household energy consumption example, \( g_j \) is simply an identity function (i.e. \( W = X \)). In many cases, sharing \( W \) instead of \( X \) does not resolve the issue of privacy since \( g_j \)'s are simple reversible functions making \( X \) discoverable from any given \( W \).

Since the goal of our system is to calculate \( \eta \) for the whole community, a simple idea is that each user computes the \( \eta \) using its local information and only shares the regression coefficients. There is no way of reconstructing the values of \( X \) and \( Y \) only by knowing the function relating them. However, it is also impossible to combine the partially-calculated models (\( \eta \)'s) and obtain the global model without extra information.

What is needed are the features that represent correlations between different attributes. Our idea is that a correlation matrix reveals very limited information about the data trace but has enough information to be used for regression modeling. Let \( X_u \) and \( Y_u \) represent the data corresponding to each user \( u \). Let \( W_u \) be the predictor matrix corresponding to user \( u \). We define the neutral feature matrices of the data collected by user \( u \) as follows:

- \( \rho_u = Y_u^T Y_u \)
- \( \psi_u = W_u^T Y_u \)
- \( \Theta_u = W_u^T W_u \)

We first observe that none of the matrices used in the above definition depends on the number of samples the user has collected, the same amount of data is sent to the server for modeling. As we see later in Section 5 this property helps enabling users’ privacy. Moreover, this simple property enables users to achieve a personalized level of privacy. A user who shares more data samples in a single transaction achieves a higher level of privacy.

The computational cost associated with extracting the feature matrices is proportional to the square of the number of samples and number of variables, \( n^2 \) and \( k^2 \). Note that the number of samples shared by a user can be as large as \( n \) in the worst-case computation analysis. Calculating \( \Theta_u \) is the most
Table 1. A sample segmented data set.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Jul.</td>
<td>1.250</td>
<td>2.5</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td>Aug.</td>
<td>0.870</td>
<td>3.9</td>
<td>72</td>
<td>73</td>
</tr>
<tr>
<td>Sept.</td>
<td>1.00</td>
<td>1.5</td>
<td>72</td>
<td>70</td>
</tr>
<tr>
<td>Oct.</td>
<td>1.45</td>
<td>1.2</td>
<td>71</td>
<td>56</td>
</tr>
<tr>
<td>Nov.</td>
<td>2.1</td>
<td>3.4</td>
<td>70</td>
<td>44</td>
</tr>
<tr>
<td>Dec.</td>
<td>2.75</td>
<td>2.3</td>
<td>70</td>
<td>26</td>
</tr>
</tbody>
</table>

computationally expensive transformation requiring \(O(k^2n^2)\) operations. Calculating \(\rho_u\) and \(\eta_u\) requires \(O(n^2)\) and \(O(kn^2)\) operations respectively. The data segmentation is linear in terms of the number of segments \(n\) and therefore is dominated by feature extraction process.

The following example illustrates how the feature matrices are obtained from the data and shared.

**Example 1 (Features).** Given the segmented data in Table 1 the client calculates the following feature matrices:

\[
\rho_u = \begin{bmatrix} 1.23 & 0.87 & 1.00 & 1.45 & 2.10 & 2.75 \\ 1.23 & 0.87 & 1.00 & 1.45 & 2.10 & 2.75 \end{bmatrix}^T = 17.3448
\]

\[
\nu_u = \begin{bmatrix} 2.5 & 74.0 & 79.0 \\ 3.9 & 72.0 & 73.0 \\ 1.5 & 72.0 & 70.0 \\ 1.2 & 71.0 & 56.0 \\ 3.4 & 70.0 & 44.0 \\ 2.3 & 70.0 & 26.0 \end{bmatrix}
\]

\[
\Theta_u = \begin{bmatrix} 2.5 & 74.0 & 79.0 \\ 3.9 & 72.0 & 73.0 \\ 1.5 & 72.0 & 70.0 \\ 1.2 & 71.0 & 56.0 \\ 3.4 & 70.0 & 44.0 \\ 2.3 & 70.0 & 26.0 \end{bmatrix}
\]

Since the segmented data are not available to the server, we need an alternative approach to derive \(\eta\) only using the shared features. An important characteristic of the feature matrices is that they are *distributive*. One can obtain the feature matrices of a community simply by adding the feature matrices of sub-communities. Let \(u_1, \ldots, u_m\) be the \(m\) users of the participatory sensing application. Each user contributes matrices \(\rho_{u_i}, \nu_{u_i}\), and \(\Theta_{u_i}\) to the community based on its private data \(W_{u_i}\) and \(Y_{u_i}\). We can write

\[
W = [W_{u_1}| \ldots |W_{u_m}]^T \quad \text{and} \quad Y = [Y_{u_1}| \ldots |Y_{u_m}]^T.
\]

These are the complete set of sensing values collected by the users, which indeed are not available to the server. We define \(\rho, \nu, \text{ and } \Theta\) as follows:

\[
\rho = Y^T Y = [Y_{u_1}^T| \ldots |Y_{u_m}^T] \quad \text{and} \quad \nu = [\nu_{u_1}| \ldots |\nu_{u_m}]^T = \sum_{i=1}^m Y_{u_i}^T Y_{u_i} = \sum_{i=1}^m \rho_{u_i}
\]

\[
\nu = Y^T Y = [Y_{u_1}^T| \ldots |Y_{u_m}^T] \quad \text{and} \quad \nu = [\nu_{u_1}| \ldots |\nu_{u_m}]^T = \sum_{i=1}^m Y_{u_i}^T Y_{u_i} = \sum_{i=1}^m \nu_{u_i}
\]

\[
\Theta = W^T W = [W_{u_1}^T| \ldots |W_{u_m}^T] \quad \text{and} \quad \Theta = [\Theta_{u_1}| \ldots |\Theta_{u_m}]^T = \sum_{i=1}^m W_{u_i}^T W_{u_i} = \sum_{i=1}^m \Theta_{u_i}
\]

Here, \(\rho, \nu, \text{ and } \Theta\) are calculated from the shared features. These are the only information that the server use to construct a model. To do that, we can rewrite (1) and make the coefficients only depending on the feature matrices:

\[
\eta = (W^T W)^{-1} W^T Y = \Theta^{-1} \nu
\]

The above equation enables the server to calculate the regression coefficient only using the shared features by \(\eta = \Theta^{-1} \nu\). In many cases, the server also needs to calculate the regression error. Again, assuming to have access to the whole data set, the error can be derived as follows:

\[
Err = (Y - \Theta \nu)^T (Y - \Theta \nu)
\]

Similar to the derivation of \(\eta\), we need to rewrite this equation so that \(Err\) can be calculated only using \(\rho, \nu, \text{ and } \Theta\):
The above process derives the values of $\eta$ and $Err$ without requiring access to the users’ data and by only using the shared features. Meanwhile, the derivation produces exactly the same results as if having access to the raw data. Therefore, our scheme successfully achieves the first design objective that is not to impose any additional modeling error.

We can observe that the modeling process employed at the server is computationally efficient. Specifically, computing $p$, $v$, and $\Theta$ need $O(nk^2)$ operations. The reason being that each matrix addition takes at most $O(k^2)$ time (size of the largest matrix, $\Theta$) and we can have at most $n$ users in the system hence $n$ additions. Calculating the regression coefficients needs matrix inversion of a $k \times k$ matrix leading to an $O(k^3)$ computation time. Error calculation takes $O(k^3)$ to complete as well. Since $n$ is much larger than $k$ in most cases, the total computation time is dominated by $O(nk^2)$.

The server-side computation is significantly more efficient than modeling the whole raw data set where users do not use a privacy firewall (which is $O(n^3)$).

### 5 Privacy Analysis

Our approach to study how the method presented in Section 2 helps enabling privacy is to show that is not likely to reconstruct user data accurately and efficiently. We first formulate our measure of privacy as the amount of error incurred while the best reconstruction of the data is done. In particular, let $y_i$ and $w_{i1}, \ldots, w_{ik}$ be the segmented data that are transformed into the feature matrices. Let $\tilde{y}_i$ and $\tilde{w}_{i1}, \ldots, \tilde{w}_{ik}$ be the values in the reconstructed set. We define the reconstruction error for each attribute $j$ to be

**Definition 1 (Reconstruction Error).** The reconstruction error of $w_j$ is the normalized sum of squared difference between the data set and the reconstructed trace:

$$RE(w_j) = \sum_{i=1}^{n} \frac{(\tilde{w}_{ij} - w_{ij})^2}{\sigma_j^2}$$

where $\sigma_j^2$ is the variance of the variable $w_j$.

In order to measure the privacy, consider the case that no information is shared by the user. The mean of the variables (i.e. $\bar{y}_i = E[y_i], \bar{w}_{i1} = E[w_{i1}], \ldots, \bar{w}_{ik} = E[w_{ik}]$) are the best estimators when no observations are available. The resulting reconstruction error is simply:

$$RE(w_j) = \frac{\sum_{i=1}^{n} (E[w_j] - w_{ij})^2}{\sigma_j^2} = \frac{\sigma_j^2}{\sigma_j^2} = 1$$

A reconstruction is effective if the reconstruction error for some variable is less than the error resulted from using the mean value. In other words, if the reconstruction error is greater than or equal to 1, it means that the transformed matrices have almost no useful information.

### 5.1 Privacy-Enabling Properties

In order to study how our approach tries to preserve users’ privacy, we show that it is hard to accurately reconstruct the private data for user $u$ (i.e., $Y_u$ and $W_u$) from $p_u$, $v_u$, and $\Theta_u$. Given the transformed matrices and their relation to the user data, there are a set of values (or a subspace) that satisfies the relation and produces the same matrices. Since the only information available from the user are the transformed matrices, an optimal reconstruction should pick the most likely values among those which satisfy the transformation. For example, it is much more probable for a user to drive a 3000lbs vehicle rather than a 500lbs vehicle. Therefore, when both values satisfy the transformation, 3000lbs is more likely to be the accurate reconstruction. We formally define the optimal reconstruction as follows:

**Definition 3 (Optimal Reconstruction).** An Optimal Reconstruction is to find the values $Y_u$ and $W_u$ that produce the given transformed matrices $p_u$, $v_u$, $\Theta_u$ while maximizing the joint probability of observing such values.

We assume that distribution information is available to the attacker. This usually comes from publicly available knowledge about the measured variable. For example, the weight distribution of vehicles sold in the US may be a public piece of information available to the attacker. The optimal reconstruction can be formulated as the solution to the following optimization problem:

$$\max \sum_{i=1}^{n} y_i^2 = p_u$$

subject to

$$\sum_{i=1}^{n} y_i w_{ij} = v_u(j) \quad \forall 1 \leq j \leq k$$

$$\sum_{i=1}^{n} w_{ij} w_{j2} = \Theta_u(j_1, j_2) \quad \forall 1 \leq j_1, j_2 \leq k$$

where $p(y_1, w_{i1}, \ldots, w_{ik})$ is the probability of observing values $y_1, w_{i1}, \ldots, w_{ik}$ which may be known to the attacker as prior knowledge. We emphasize that likelihood maximization is a key to the reconstruction. Here, we assume that data from any two segments shared by the user are independent. This is the case because each segment can belong to an arbitrary time and location and is uncorrelated to others due to segmentation. Note that the optimal reconstruction does not directly calculate values of model inputs, $X$, but rather the values of matrix $W$. However, in many applications the exact
values of $X$ can be derived from $W$. In Section 6, we discuss the details of such derivation when doing a case study.

Another important point here is that the value of $n$ and consequently the number of variables to be reconstructed is not available to the attacker in most of the cases. To overcome this, one may try several guesses of $n$ and choose the reconstruction with the maximum likelihood. Perhaps, starting with a value of $n$ that makes the number of constraints and variables equal (i.e., $n = \frac{k}{2} + 1$) and increasing $n$ until the value of the objective function is minimized.

Next in this section, we discuss why one cannot easily reconstruct the original values from the transformed matrices. We first describe why given a large enough $n$, the reconstruction accuracy is expected to be low. Then, we argue that finding an accurate reconstruction becomes computationally expensive for large values of $n$.

### 5.1.1 Inaccuracy of Reconstruction

We first discuss the following question: If someone succeeds at finding an optimal reconstruction, how close are the reconstructed data points to the actual data trace? Intuitions to answer this question are presented below. Later, we demonstrate an example answer for a specific empirical study.

First observe that the number of constraints in the optimal reconstruction problem is $\frac{k^2 + 3k}{2} + 1$. When the number of data points $n(k + 1)$ is less than the number of constraints, only a single feasible assignment exits. Hence, the reconstruction may be performed with 100% accuracy and no privacy is preserved. On the other hand, when $n$ tends to infinity, the feasible solution space becomes more and more relaxed as the number of constraints remain constant. In this case, the optimal solution converges to $(E[y], E[w_1], \ldots, E[w_k])$. The reconstruction error tends to 1, using a derivation similar to (4). This means that for large enough $n$, it becomes unlikely to reconstruct the private data.

### 5.1.2 Inefficiency of Reconstruction

Next, we give intuitions on why optimally reconstructing data as in (5) becomes computationally expensive for large values of $n$ and $k$. We emphasize that our discussion is the worst-case complexity analysis for finding the optimal solution and merely gives a hint regarding the complexity of reconstruction in general. The analysis does not hold for the cases where a non-optimal solution can still be a threat to privacy, where the particular form of features allows for fast computation, or where the values of $n$ and $k$ are always small.

It is obvious that the objective probability distribution should be approximated by a well defined smooth function. Otherwise, the optimal solution cannot be obtained in less than an exponential time even without any constraints. Let us assume that the objective function has a very simple form (e.g., linear or convex quadratic). Still, the constraint space is not convex since the equality constraints are not affine. This means that there is no hope to solve such a problem through convex optimization methods. On the other hand, any general non-convex quadratically constrained program is shown to be NP-Hard [37]. In fact, any integer linear program can be written as a quadratically constrained optimization problem. Therefore, an optimal reconstruction of the user data trace takes an exponential amount of time with respect to the number of variables, $n(k + 1)$, unless $P = NP$.

### 5.2 Conditions to Protect Privacy

It is important for the privacy firewall to identify the conditions where the privacy of the user data is not protected at all. In particular, this happens when the number of segments used to create features is small. Here, we experimentally study the privacy-enabling properties of feature matrices to derive a heuristic lower bound on the number of segments to be used in the client. The client application simply warns the user if the number of samples used for the transformation is less than the threshold.

Our first step in order to evaluate privacy is to approximate the reconstruction process by using a heuristic model of the objective function. Our heuristic is to assume that the maximum likelihood is obtained when the solution is close to the expected value of each parameter’s marginal distribution. Therefore, we minimize the distance of each variable from the expected value of its corresponding distribution. Let $y$ and $w_i$ be the random variables corresponding to the output attribute and the predictor variables respectively.

$$\min \sum_{i=1}^{n} \left[ (y_i - E[y])^2 + (w_{i1} - E[w_1])^2 + \ldots + (w_{ik} - E[w_k])^2 \right]$$

$$\sum_{i=1}^{n} y_i^2 = \rho_u$$

$$\sum_{i=1}^{n} y_i w_{ji} = v_u(j) \quad \forall 1 \leq j \leq k$$

$$\sum_{i=1}^{n} w_{ji} w_{j'i} = \Theta_u(j_1, j_2) \quad \forall 1 \leq j_1, j_2 \leq k$$

There are two methods to approximate the above optimization problem. First, the problem can be relaxed into a semi-definite program (SDP) and then the solution of the SDP converted to a feasible solution for our quadratically constrained program. However, in most cases, SDP relaxations needs a specific form of constraints. The second approach is to use a non-linear optimizer based on interior-point barrier or sequential quadratic programming. We use
KNIITRO [28] non-linear solver package in our implementation. This package can be linked to a MATLAB code.

Using our MATLAB implementation, we evaluate the time efficiency of this non-linear solver with respect to the number of samples used for the transformation. Here, we simply assume that we know the value of $n$ for the reconstruction. We experiment with various number of predictors (model inputs), $k$, ranging between 3 to 5. For any given $n$ and $k$, we divide the whole data set into groups of $n$ samples. For each group, we perform the transformation and reconstruction. At the end, we report the average number of iterations over all of the groups.

As Figure 2 suggests, there is a peak in the reconstruction time for any given value of $k$. This is the situation where the number of variables, $n(k+1)$, is closest to the number of constraints, $\frac{k^2 + 3k}{2} + 1$. For smaller values of $n$, the number of constraints is more than the free variables. A single feasible solution in this case may be the reconstruction result. On the other hand, for larger values of $n$, the number of free variables is more than the constraints. It is easier then to find a point closer to the mean to maximize the objective function. Based on this we expect the reconstruction error to be very high after $n = k$.

We should note that since the rows of $W$ and $Y$ are not ordered, the order of the reconstructed matrices $\tilde{W}$ and $\tilde{Y}$ may be different from the original data. In our implementation, we try all permutations of the rows to find the best match. This, of course, is only feasible for small values of $n$, and is just to show that even the best possible reconstruction will not be accurate against the feature matrices.

To empirically demonstrate that accurate reconstruction is unlikely for when $n$ is larger than a threshold and to find that threshold, we generate random input and output values with various correlations. In particular, when no correlation is present, we generate 1000 different data segments with $k = 4$ using independent random values distributed with a normal distribution. We call this distribution the case of no correlation. When correlations are present, we generate random input:

$$W = RU$$

$$W^T W = U^T R^T RU = U^T U = C$$

The last equation comes from the fact that $R$ is a matrix of independent variables and $R^T R = I$. We simply use several correlation matrices and derive $U$ using a Cholesky decomposition. This is an established method of adding correlation to independent random variables. Finally, we create the highest possible vertical correlation by making each attribute $w_i$ be $w_i + c$ where $c$ is a constant offset.

In the first experiment, we change the value of $n$ used to create the feature matrices and derive the average reconstruction error for 10 different input matrices with various correlations. As Figure 3 suggests, even in the extreme case of the highest vertical correlation, the reconstruction error is close to 1 when $n > 2k$.

In the next experiment, we try various horizontal correlations while employing a medium vertical correlation. Similarly the highest correlation is obtained when the segments are just a shifted version of each other by a constant offset. We should emphasize that our data segmentation is required to remove any significant horizontal correlation. This experiment is to show what happens if residual temporal correlations exist between segments. Results in Figure 4 show that for $n > 2k$, the error approaches 1 indicating privacy protection against this particular reconstruction.

Figures 3 and 4 suggest that the user can personalize the amount of privacy that is guaranteed based on the number of segments shared. One simple way to implement this is to have multiple threshold values on the number of samples being shared. Each threshold guarantees a higher degree of privacy (a higher minimum reconstruction error).

To derive the relation between a safe value of $n$ and $k$, in the last experiment we change the value of $k$ and create correlated input samples. Figure 5 plots the minimum number of samples required to guarantee a reconstruction error of at least 0.9. Observe that $n = 2k$ is always a safe bet. The curve suggests that the client-side software should compute neutral
features based on a number of segments that is at least twice the number of model inputs.

The authors admit that this rule of thumb is not supported by rigorous theoretical analysis. A formal investigation is beyond the scope of this paper. Here, the conjecture is merely observed without formal proof. In general, however, the client-side software, when asked to share data, can compute the neutral features given the currently available segments then perform the reconstruction itself and estimate reconstruction error. If the error is significantly less than 1, the software may warn the user that privacy may be breached.

6 A Case Study

In this section, we implement our proposed scheme and evaluate it on a participatory data set used for green navigation [21]. In that service, individual drivers install OBD-II scanners in their cars which record the engine sensor parameters along with GPS location information on SD cards. Their service then uses regression modeling to suggest the most fuel-efficient routes between arbitrary source destination pairs [19] by using a prediction model to estimate the car’s expected fuel consumption on each road segment.

There are several privacy-preserving techniques proposed for participatory sensing applications that can be comparable in that they rely on data alteration. Mostly they propose specific noise models to overcome correlations in the data set [22,17,34,20]. Since we have already removed the correlations between data points during segmentation, we compare our technique to one that shares raw segments perturbed by white noise. This simple technique in fact outperforms more complex correlated noise schemes in terms of impact on the final model. We first present the details of our implemented client and modeling server. Next, we compare the level of privacy achieved by our approach to perturbation-based techniques. Finally, we compare the accuracy of modeling of our approach and perturbation-based approaches.

6.1 Implementation

We implemented a desktop client and server as C++ libraries. The client program takes the data trace file and a configuration file as input. Each measurement attribute is a column in the trace file. The configuration file stores the settings of the application: i) A unique application id, ii) the segmentation interval, iii) the segmentation attributes (e.g., time), and iv) predictor functions that map X to W (f). The application id is used for later correspondence with the server to make sure that the feature matrices are used for the correct model. The segmentation interval and segmentation attribute are used by the client to segment the input trace. In particular, the client reads the input trace file row by row and groups the samples based on the segmentation attributes. There can be multiple attributes that are used as the segmentation attribute. For example, location traces from a GPS receiver consists of a longitude and a latitude. The segmentation process simply takes the euclidean distance between the segmentation attributes of two consecutive samples. The increments are then added and when the sum reaches the segmentation interval, this batch of samples are used to produce one segment. All samples for each attribute are added and divided by the segmentation interval to get the average value to be used in the segmented data. These values correspond to x_i in our notation.

Next, the client checks for the privacy condition and if n < 2k, warns the user to use a larger input. The next step in the client is to derive the values of w_i from x_i. This is done using the configuration file settings. The client calculates \( \rho, v, \) and \( \theta \). The feature matrices are transferred over a TCP connection using XML. The XML message contains an application id and the values of matrices \( \rho, \theta, v \).

The server program maintains a list of models mapped to the application ids. Each model consists of the aggregate values of \( \rho, v, \) and \( \theta \). When an XML message is received from a client, the application id is looked up to find the corresponding model. The feature matrices in the XML are simply added to the aggregate matrices. The client configuration file ensures the same ordering of columns when calculating the matrices. The server derives the model coefficients using Equation (2) and writes them to a file.

6.2 Experimental Setup and Data Set

To evaluate our privacy-aware mechanism, we utilize data from the experimental deployment of a green navigation system. This data consists of geo-tagged engine sensor measurements for a range of vehicles and constitutes a total of over 1000 miles of driving. A total of sixteen users (with different cars) drive over the course of three months [21].

After segmentation, the total number of data segments in the data set are 650. Each segment represents an approximately 2 miles of driving data, although shorter segments are present when routes where shorter than 2 miles or were not a multiple of that distance. There are 5 parameters for each segment, which are inputs to (and output of) a model of vehicular fuel consumption. These are: i) fuel consumption over the segment, \( y = f \), ii) \( w_1 = m(ST + vTL) \) where \( m \) and \( v \) are the mass and the average velocity of the vehicle, \( ST \) and \( TL \) are the number of stop signs and traffic lights encountered, iii) \( w_3 = m^2 \), iv) \( w_2 = m \), and v) \( w_4 = A \) where \( A \) is the frontal area of the car. Using our notation, we can write \( x_1 = m, x_2 = v, x_3 = A, x_4 = ST, x_5 = TL \). Therefore, we have \( k = 4 \) and \( d = 5 \).

The values of \( m, v, A \) can easily be obtained by knowing parameters \( w_2, w_3, \) and \( w_4 \). The values of \( ST \) and \( TL \) can also be determined since they are integers and the number.
This makes 1 output attribute and 4 predictors, hence In our experiments, we use all the five parameters by default. In some experiments, we drop the last parameter from the parameters for different values of \( n \). In the first experiment, we compare the reconstruction error resulting from using the two approaches when the number of samples shared by each user, \( n \), varies. The total number of segments are the constant, 650. This divided by \( n \) is the number of users who share the data. For every \( n \), we average the resulting reconstruction over all users data and report the results in Figure\[7\]. We do this for both \( k = 3 \) and \( k = 4 \). The result from the perturbation scheme does neither change with \( k \) nor with \( n \). Therefore, we only present a single line.

Next, we repeat the same experiment while studying the reconstruction error of each individual parameter \( x_k \). Using all of the predictor variables we report the average reconstruction error of each parameter in Figure\[8(a)\]. The results show that reconstruction errors of 0.8 and 1.0 are achieved for all parameters when \( n = k = 4 \) and \( n \geq 2k = 8 \) respectively. Figure\[8(b)\] repeats the same experiment and reports the error corresponding to the most accurate reconstruction among different samples instead of the average. This is the worst case privacy for in a particular set of data points. The results still show the complete privacy protection for \( n \geq 2k \).

In order to evaluate the effect of the number of predictors on privacy, we also evaluate individual parameter reconstruction errors for \( k = 3 \), by dropping the last predictor variable from the data set. Figures\[9(a)\] and \[9(b)\] shows the average and worst-case privacy obtained by using our approach. The result verifies that a privacy-breach is unlikely if the number of samples is larger than the heuristic threshold.

### 6.4 Prediction Accuracy

In this set of experiments, we evaluate the prediction accuracy of the constructed model at the server and compare it against a perturbation scheme. To this end, we perform a cross validation by leaving out a single segment from the data set and use the rest to construct the model which is then used to predict fuel consumption of the segment in question. To emulate multiple users, we divide (the rest of) the data set into groups of 10 segments and share them using our privacy-aware client firewalls. The regression coefficients computed on the server are then used to estimate the fuel consumption on the single segment that was left out. We add the estimated fuel consumption values on all segments to obtain a total fuel estimate. We calculate the relative error (in percents) between this estimated value and the sum of actual fuel consumption values. This calculation is called cumulative error. To calculate the prediction error of the perturbation scheme, we use the same cross-validation and error calculation scheme, except that the model is computed from individually shared segments perturbed with additive noise. Thus, to estimate fuel consumption of a segment, we simply add white noise (of a given energy) to the data set with one

![Figure 6. Distribution of various parameters in the green navigation data set.](image)

![Figure 7. Comparing our approach against data perturbation in terms of the average reconstruction error of all parameters for different values of \( k \).](image)
segment removed and calculate the regression coefficients of the noisy data. The power consumption for that segment is then estimated from the obtained model and the process is repeated for all other segments in turn.

Our first experiment to evaluate the prediction accuracy compares the two schemes when the amount of noise energy used for perturbation changes. Figure 10 shows the results where the prediction error is represented using the mean squared error between the estimates and the actual fuel consumption values where all values in the dataset are normalized to $[-1, +1]$ range. Note that, the prediction accuracy of our approach is presented as a reference and remains constant since we do not use any noise. The $x$-axis shows the ratio of the noise variance to the variance of data parameters. Note that, prediction error linearly changes with noise energy. However, it is clear from the figure that even with low noise energy values, the prediction error of the perturbation scheme is much higher than ours. This is despite the fact that our scheme is always providing privacy.

The next experiment studies the effect of the total number of segments that are collected for modeling. We fix $n = 10$ and noise variance ratio to be 1 and change the total number of available segments from 10 to 500. Figure 11 depicts that the prediction error decreases as the number of samples increases. This is simply because the constructed model becomes more accurate. However, when the number of segments is very small, the perturbation scheme shows an additional prediction error because the noise values do not cancel out when the number of samples are small.

In the final experiment, we evaluate the prediction accuracy of the fuel consumption for each individual car and report it in Table 2. For each car, only the data set that corresponds to the car is used for the prediction. The segments are similarly left out one by one for cross-validation. This result simply shows that the prediction error increases at least by four times when using perturbation. In the green navigation service which relies on accurate prediction of energy consumed on various segments in order to compute the most fuel-efficient routes and where different routes often differ in less than 5% to 6% of the total fuel consumption, 2% prediction error certainly affects the routing decisions made by the service. Our scheme on the other hand, enables the application to have the same accuracy as that achieved in the absence of privacy-enabling techniques.

7 Discussion

In the previous sections, we discussed how our technique matched the two design goals presented in the beginning. Al-
The privacy argument presented in the paper has two major shortcomings: i) The quantification of privacy does not capture all forms of privacy breaches, and ii) this scheme has not been strictly shown to guarantee privacy, as defined by our measure of privacy. This section discusses these limitations in detail and presents the cases where the approach may perform poorly to satisfy the neutrality condition.

Our quantification of privacy captures a specific form of information about the user, i.e. the exact data point values in users’ data traces. Sometimes, more high-level information about the user can be deduced without reconstructing the exact data point values. For example, the magnitude of values in the feature matrix may reveal information about the range of values in the original trace. Changes in the correlations can also be used to infer information about how and when a user changes behavior (e.g. in energy consumption). To reflect those concerns, various definitions including Bayes-Optimal and differential privacy have been studied in the literature [14, 9]. Understanding the performance of our scheme under such notions helps a more concrete analysis of the privacy properties.

Based on our privacy-measure, we identify two major factors that can affect the possibility of revealing data after applying the privacy firewall. First, the distribution of the original data: a discrete and narrow distribution leaves a fewer number of choices for the reconstruction. Narrow distributions means that the value of a variable is approximately known to the attacker even before sharing any data. It can be the case that some data points are estimated rather accurately while the total reconstruction error is still high.

The second factor is the correlation values: For example, if the correlation values are all 1s, the whole data trace is a constant and therefore can be reconstructed perfectly if the average value for the user is revealed. Generally, a higher correlation makes the process of reconstructing the variables easier as the search space shrinks.

### 8 Related Work

In recent years, a number of techniques have been proposed for modifying or transforming data in such a way as to preserve privacy. Such methods can be classified into four main categories described in detail below.

First, randomization techniques that add noise to the original data points have been used to hide the real value of sensitive data and other attributes (e.g., the trend of the data over time) [2, 6]. They traditionally distort data for methods such as surveys which have an evasive answer bias because of privacy concerns [40, 31]. Fuller [18] and Kim and Winkler [27] showed that some simple statistical information (e.g., means and correlations) can be preserved by adding random noise. In [4, 3], independent random noise (e.g., Gaussian) with high enough power is used to perturb user data. However, high noise power might decrease the utility of the shared data as well and the authors do not quantify this trade off. Recently, Ganti et al. [20] proposed that correlated noise, which has the same distribution as real data, can be used to perturb time-series data. This perturbation method is resilient to traditional filtering techniques, such as Kalman filter [24], and Spectral filtering [25]. Using randomization techniques in the context of privacy preserving regression, however, will introduce error in the regression model.

Another set of randomization techniques preserve differential privacy using randomized aggregation functions [14, 9, 36]. When an aggregate value is derived by a trustworthy entity or the user client, differential privacy is preserved if adding or removing a data item does not significantly change the output (aggregate) probability distribution. Like data point perturbation, differential privacy methods rely on randomization that introduces noise to the regression model.

Second, the k-anonymity model [30] was developed because of the possibility of indirect identification of records.
from public databases. For example, the identity of a patient can be inferred from their home address or cellphone number. In the k-anonymity method, the granularity of data is reduced using techniques such as generalization and suppression. The l-diversity model [33] was designed to handle some weaknesses in the k-anonymity model including the cases where there is homogeneity of the sensitive values within a group. Many variants of the above methods exist in current literature. A good survey of the corresponding algorithms may be found in [10]. Although k-anonymity is widely used to hide user identity in large database, it can not be applied into this problem because useful information for regression will be lost during generalization and suppression.

Distributed privacy preservation [42,55] is used to derive aggregate results from data sets which are partitioned across these entities. While the individual may not desire to share their entire data set, they may consent to limited information sharing with the use of a variety of protocols. The overall effect of such methods is to maintain privacy for each individual, while allowing the aggregate results to be correctly computed over an entire group. Our proposed technique in this paper falls under this category. For this purpose, the data sets may either be horizontally partitioned or be vertically partitioned. In horizontal partitioning [11,12,32], the individual records are spread out across multiple entities, each of which have the same set of attributes. In vertically partitioned data sets [11,39], the individual entities have different attributes of the same set of records. There have been also a lot of efforts in finding good privacy preserving regression methods [38,13,26]. None of them, however, quantify the achieved privacy.

Finally, several recent works have proposed cryptographic solutions for privacy-preserving modeling or aggregation of distributed data [38,15,26]. Some of these techniques, like our technique, do not perturb data and hence enable exact computation of certain aggregate functions. These solutions allow data sources to publish data in encrypted form, which hides the raw data values. They generally use homomorphic encryption so that the central server can compute aggregate functions directly on encrypted values. However, such encryptions are extremely expensive (up to 30× more expensive than regular encryption) and only a small class of functions can be computed on such encrypted data. Our solution is computationally cheaper and more general than these cryptographic solutions.

Some previous work study how to provide different levels of privacy to different users. Xiao and Tao [41] proposed the notion of personalized privacy to allow each data owner to specify her own privacy level. However, the technique works in a centralized setting and for string attributes organized in a taxonomy tree. In contrast, the technique in this paper is targeted towards numerical data in a distributed setting.

9 Conclusion

This paper presented a privacy-aware scheme that enables regression modeling of participatory sensing data. We consider participatory sensing applications that develop general prediction models from data. The novelty of this work comes from the fact that it constructs regression models exactly the same as if private data where available to the server.

The main idea of the paper is to transform user data traces into neutral features that can be shared with the community and with the aggregation server while minimizing the threat to privacy. The time-series traces that are sensed by the user are first segmented into intervals of larger scale. The next step is to convert the segmented data into matrices that give little insight into the original data.

We show how given the transformed matrices, a server can construct the same regression model as if it has access to the data traces. The privacy-enabling properties of our transformation technique are studied. Intuitive arguments and empirical evidence suggest that when a large enough number of samples is transformed into a single feature and shared by a user, a privacy-breath is unlikely. Also, we show how the a personalized level of privacy is achieved by varying the number of samples included in the neutral features.

A data set from a green navigation service is used as a case study. We implement our approach as a client library and a server that enables privacy-enabling model construction in this application. Experimental results show that our technique does not change the accuracy of navigation while trying to make the user data private. In comparison, traditional data perturbation schemes introduce a significant amount of prediction error.

Future work on this topic will include a more formal quantification of privacy properties and their dependency on the user’s data set, the number of segments used, and the number of model inputs available. Deployment data will be collected from example applications to investigate usability issues and issue of client interface. Finally, we shall investigate extensions of this approach to other modeling frameworks, besides linear regression.

10 Acknowledgments

This work was funded in part by NSF Grant 1040380. We thank the shepherd, Dr. Phillip Gibbons, and the anonymous reviewers for providing valuable feedback.

11 References


